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THE RADIUS AND ELLIPTICITY OF URANUS FROM ITS OCCULTATION OF SAO 158687

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ABSTRACT

From occultation timings obtained from the Kuiper Airborne Observatory and from Cape Town for the 1977 March 10 occultation of SAO 158687 by Uranus, the equatorial radius, R_e, of the planet has been determined to be 26,228 ± 30 km and its ellipticity $\epsilon = 1 - R_p/R_e = 0.033 \pm 0.007$. These values refer to the 1.0×10^{14} cm^{-3} number-density level, under the assumption that the upper atmosphere is composed of H_2 and He with a mean molecular weight $\mu = 2.20$. The dominant source of uncertainty is the position of the center of the ring system, which was used to define the center of Uranus in our analysis. A rotation rate of 12.8 ± 1.7 hours for the planet is implied by our value for the ellipticity, under the assumption that Uranus is in hydrostatic equilibrium below the 1.0×10^{14} cm^{-3} number density level.

Subject headings: occultations — planets: Uranus

I. INTRODUCTION

During the 1977 March 10 occultation of SAO 158687 by Uranus, two chords across the planet were obtained: one from the Kuiper Airborne Observatory (KAO) and the other from Cape Town. These chords were only about 1000 km apart, which would normally be inadequate for obtaining a reliable radius and ellipticity for a planet as large as Uranus. However, we can use the center of the ring system (Elliot *et al.* 1978) to define the center of the planet, eliminating two unknown parameters from the least squares solution and allowing precise values for the radius and ellipticity to be obtained. From the ellipticity and the zonal harmonic coefficient, J_2 (obtained from the precession of the ϵ ring, Nicholson *et al.* 1978), we can also obtain the rotation rate of Uranus, if we assume that the planet is in hydrostatic equilibrium.

II. OBSERVATIONS AND SKY-PLANE COORDINATES

Descriptions of the procedures used for observing the occultations of SAO 158687 by Uranus and its rings have been given previously (Elliot, Dunham, and Mink 1977; Elliot *et al.* 1978; Churms, Elliot, and Dunham 1980; Dunham, Elliot, and Gierasch 1980). We have chosen to refer our radius to the 1.0×10^{14} cm^{-3} number-density level in the atmosphere, rather than the "$\frac{1}{2}$ light" level used in previous analyses for other planets: we have done this because the $\frac{1}{2}$ light level will not refer to the same number-density level in the atmosphere for different occultations if the Earth-Uranus distance is different (see Elliot 1979 for a review). However, we have also included our analyses for the $\frac{1}{2}$ light and 5×10^{14} cm^{-3} number-density levels as an indicator of the consistency of our results.

Accordingly, we have obtained the times when 1.0×10^{14} cm^{-3} and 5.0×10^{14} cm^{-3} number-density levels were reached for the immersion and emersion data of the KAO and of Cape Town from the reduction of the number-density profiles of the atmospheric data (Dunham, Elliot, and Gierasch 1980). In these reductions we assumed that the atmosphere is composed of H_2 and He, with a mean molecular weight $\mu = 2.20$. The resulting values for the refractivity of the atmosphere at STP are given in Table 2 of Dunham, Elliot, and Gierasch (1980). The times for $\frac{1}{2}$ light were obtained by fitting an occultation curve appropriate for an isothermal atmosphere (French, Elliot, and Gierasch 1978) to the data. The occultation times are given in Table 1. For the KAO the times represent an average for the three data channels, and we have also given the coordinates of the KAO at these times. The locations of the occultation points on Uranus for $\frac{1}{2}$ light are shown in Figure 1 of Elliot *et al.* (1978).

The data reduction procedures followed those used by Elliot *et al.* (1978), except as noted. First, the linear coordinates in the sky plane (ξ', η') were obtained from equations (1) and (2) of that paper. Corrections of 2.64×10^{-4} s in right ascension and $-1''.35154$ in declination were added to the ephemeris of Uranus (Table 4 of Elliot *et al.* 1978) so that the resulting values of (ξ', η') would be (0.0, 0.0) at our adopted center of the ring system (solution [2] in Table 6 of Elliot *et al.* 1978).

TABLE 1

OCCULTATION TIMES AND COORDINATES

A. KUIPER AIRBORNE OBSERVATORY

Level in the Atmosphere (molecules cm^{-3})	Time (UTC)	Latitude (South)	Longitude (East)	Correction for Refraction (km)	Correction for Gravitational Bending of Light (km)	ξ (corrected, km)	η (corrected, km)
$\frac{1}{2}$ light ($\sim 7 \times 10^{13}$)....	20:53:27.98	51°27′.0	89°27′.2	54.8	26.4	−1,023.5	26,221.2
	21:18:08.76	51°25′.5	94°44′.0	52.7	26.4	+15,771.2	20,877.3
1.0×10^{14}...........	20:53:29.41	51°27′.1	89°29′.8	93.2	26.4	−895.1	26,218.6
	21:17:56.98	51°25′.7	94°41′.5	82.2	26.5	+15,655.4	20,943.4
5.0×10^{14}...........	21:55:01.79	51°28′.3	89°45′.9	307.1	26.7	+38.9	26,135.7
	21:16:22.97	51°27′.2	94°22′.0	353.2	26.8	+14,742.3	21,507.1

B. CAPE TOWN

Level in the Atmosphere (molecules cm^{-3})	Time (UTC)	Correction for Refraction (km)	Correction for Gravitational Bending of Light (km)	ξ (corrected, km)	η (corrected, km)
$\frac{1}{2}$ light ($\sim 7 \times 10^{13}$)......	20:57:21.82	46.9	26.4	−3,130.1	26,042.4
	21:28:58.56	49.0	26.5	+17,556.7	19,369.4
1.0×10^{14}.............	20:57:35.44	83.9	26.4	−2,995.3	26,032.5
	21:28:46.51	85.9	26.5	+17,449.2	19,439.6
5.0×10^{14}.............	20:58:44.96	336.6	26.8	−2,259.7	26,040.2
	21:27:30.05	353.8	26.9	+16,785.4	19,915.5

Two corrections must be added to the sky-plane coordinates (ξ', η') observed at the Earth to obtain the coordinates (ξ, η) at Uranus through which a light ray would have passed to arrive at (ξ', η'). The (ξ, η)-plane has its origin at the center of Uranus and lies parallel to the (ξ', η')-plane, which passes through the center of the Earth. First, we must correct for refraction by the Uranian atmosphere by adding an amount $D\theta$, where D is the Earth-Uranus distance and θ is the instantaneous refraction angle determined from the numerical inversion of the light curves (French, Elliot, and Gierasch 1978). Second, we must add an amount $D\Delta\theta$ to correct for the general relativistic bending of light by Uranus (see eq. [8] of Elliot *et al.* 1978). These corrections are given in Table 1 and were added to $(\xi'^2 + \eta'^2)^{1/2}$ to obtain the values of $(\xi^2 + \eta^2)^{1/2}$. The position angle for each occultation point, $\theta_{\xi\eta}$, was set equal to its value $\theta_{\xi'\eta'}$ in the (ξ', η')-plane.

The procedure used here to obtain (ξ, η) for the occultation points should introduce deviations of only a few tenths of a kilometer from the rigorously correct method of successive iterations of the bending-of-light correction, the numerical inversion process, and a solution for the ellipticity of Uranus. The rigorous method was not employed because of its complexity, since its additional accuracy was not required.

III. FITS FOR RADII AND ELLIPTICITIES

We can write an equation that should be satisfied by the observed data points (ξ, η) in terms of the equatorial radius of Uranus, R_e, and its ellipticity, $\epsilon = (R_e - R_p)/R_e$, where R_p is the polar radius of the planet. First, we express (ξ, η) in polar coordinates, so that $r_{\xi\eta} = (\xi^2 + \eta^2)^{1/2}$, $\xi = r_{\xi\eta} \sin \theta_{\xi\eta}$, and $\eta = r_{\xi\eta} \cos \theta_{\xi\eta}$. Then, if B is the uranocentric declination of the Earth and P the position angle of the north pole of Uranus in the (ξ, η)-plane, then

$$r_{\xi\eta}(\theta_{\xi\eta}) = R_e \left\{ \frac{\sin^2 B + (1 - \epsilon)^2 \cos^2 B}{\cos^2(\theta_{\xi\eta} - P) + [\sin^2 B + (1 - \epsilon)^2 \cos^2 B] \sin^2(\theta_{\xi\eta} - P)} \right\}^{1/2} . \quad (1)$$

Values of R_e and ϵ were found by fitting equation (1) by least squares to the four data points for each of the three number-density levels: 1.0×10^{14}, 5.0×10^{14} cm^{-3}, and $\frac{1}{2}$ light, which corresponds to a number-density level of about 7×10^{13} cm^{-3}. The angles P and B, derived from the coordinates of the pole of the satellite plane (Dunham 1971), were set at the values $-84.^\circ 61259$ and $+53.^\circ 46174$. The results of these fits, along with the formal errors in the fitted quantities R_e and ϵ, are given in the top line of Table 2. The correlation coefficient between R_e and ϵ was about 0.77, and the rms deviation (per degree of freedom) of the values of $R_{\xi\eta}$ from the fit was 10.9 km in the solution for the 1.0×10^{14} number-density level.

TABLE 2

FITTED EQUATORIAL RADII AND ELLIPTICITIES

Offset from Adopted Center of Ring System ($\Delta\xi$, $\Delta\eta$ in km)	Number Density Level (molecules cm^{-3}): "$\frac{1}{2}$ light" ($\sim 7 \times 10^{13}$)	1.0×10^{14}	5.0×10^{14}
Common	26,245.1 ± 1.0	26,228.4 ± 8.5	26,142.6 ± 3.7
(0, 0)	0.0310 ± 0.0005	0.0329 ± 0.0040	0.0325 ± 0.0020
α Ring	26,213.5 ± 2.0	26,196.6 ± 7.5	26,109.4 ± 5.5
(28, 35)	0.0354 ± 0.0009	0.0373 ± 0.0036	0.0368 ± 0.0029
β Ring	26,239.1 ± 1.7	26,222.5 ± 9.1	26,137.5 ± 2.9
(−13, 4)	0.0272 ± 0.0008	0.0291 ± 0.0044	0.0288 ± 0.0015
γ Ring	26,259.5 ± 1.2	26,242.8 ± 8.7	26,157.3 ± 3.4
(−6, −15)	0.0307 ± 0.0006	0.0326 ± 0.0041	0.0322 ± 0.0018
δ Ring	26,274.1 ± 1.4	26,257.4 ± 8.8	26,172.1 ± 3.2
(−11, −30)	0.0307 ± 0.0006	0.0326 ± 0.0041	0.0322 ± 0.0017

We note that the values of ϵ are nearly equal for the three different levels in the atmosphere. This agreement shows that the data give the same value of the ellipticity for different definitions of a reference surface, which is important to establish. However, we do not consider the three analyses to be independent, in the sense that we could reduce the error in the ellipticity by averaging the results for the three levels.

The formal errors in R_e and ϵ have somewhat different magnitudes for the different levels, an effect we attribute to the statistics of small numbers (i.e., we are fitting two parameters to four data points for each case). With such a small number of data points there could be a significant "error in the error estimate," and we have no way of knowing the distribution function of the errors. However, two factors mitigate the importance of these uncertainties: (*a*) we have chosen the level with the largest formal errors and (*b*) our final error estimates have been substantially increased to account for possible systematic errors (see discussion below).

An obvious source of systematic error in the values of R_e and ϵ is the uncertainty in the actual position of the center of the Uranian ring system (Elliot *et al.* 1978). A rigorous standard deviation for position of the ring-system center is not presently available, so we have investigated the possible size of this effect by solving for R_e and ϵ using the centers for the individual rings α, β, γ, and δ obtained by Elliot *et al.* (1978). The results of these fits are also given in Table 2. We note that again the ellipticities are nearly equal for a given adopted center, but changing the adopted ring center changes the values of R_e and ϵ by amounts greater than their formal errors.

Other possible sources of systematic error had lesser effects. For example, solving for R_e and ϵ with the KAO data alone and the Cape Town data alone yielded solutions that differed from that with all data together by $\sim$0.0008 for ϵ and 1.5 km for R_e. We also changed the values of P and B by $\pm 0\overset{\circ}{.}05$ (about $1\frac{1}{2}$ times their formal errors given by Dunham 1971) and found that the fitted values of ϵ and R_e changed by ± 0.0002 and ± 0.2 km. The wavelike temperature variations in the Uranian atmosphere (Elliot and Dunham 1979; Churms, Elliot, and Dunham 1980; Dunham, Elliot, and Gierasch 1980) apparently have only a small effect on the fitted values of ϵ, since the three levels span about one cycle of the variations, yet showed good agreement in their solutions for ϵ.

If we have assumed the wrong composition for the Uranian atmosphere, our fitted value for the ellipticity would not be affected, but the equatorial radius would not refer to the 1.0×10^{14} cm^{-3} number-density level. The actual number-density level to which our solution refers could be found by scaling the actual refractivity of the Uranian atmosphere to that which we have assumed (see eq. [2] of French, Elliot, and Gierasch 1978).

As our standard solution, we have chosen to adopt the 1.0×10^{14} number-density level (with the common ring center), which yielded an equatorial radius $R_e = 26{,}228$ km and an ellipticity, $\epsilon = 0.033$. The dominant source of uncertainty in these values would appear to be our imprecise knowledge of the center of the ring system. The error in the center of the ring system is a combination of systematic and random errors (Elliot *et al.* 1978), so that our estimates of the errors in R_e and ϵ should be considered educated guesses, at best. We feel that ± 30 km for R_e and ± 0.007 for ϵ are reasonable estimates for the "one sigma" error bars (i.e., the odds are about 2:1 that the true value lies within one sigma of our adopted values).

From the values of R_e and ϵ obtained here, we note that the "central flash" (Elliot *et al.* 1977) should be observable for future occultations at a distance of $\epsilon R_e \approx 800$ km on either side of the center line. Observation of the central flash profile should allow a further check on the ellipticity as well as an estimate of the extinction of the Uranian atmosphere—such as we obtained for Mars from the ϵ Geminorum occultation data (Elliot *et al.* 1977).

IV. RADIUS OF THE VISIBLE DISK

To compare our results with equatorial radii determined by other methods, we must find the altitude difference between the 1.0×10^{14} number-density level and the visible disk. We shall define the radius of the visible disk as that level in the atmosphere for which the optical depth, τ, for visible light is 1.0 at the limb. However, a complication immediately arises: is the extinction at the $\tau = 1.0$ level dominated by thin clouds, aerosols, or Rayleigh scattering? The answer to this question is not readily available, so we shall begin by calculating the radius for which $\tau = 1.0$ for Rayleigh scattering.

We can write an equation for τ_R, the optical depth due to Rayleigh scattering for a slant path through the atmosphere, if we define the following quantities: R, the radius for which the path lies parallel to the limb; n, the number density of the atmosphere at radius R; H, the number-density scale height of the atmosphere; λ, the wavelength of the light; L, Loschmidt's number; ν_{STP}, the refractivity of the atmosphere at STP; and p_n, the depolarization factor (McCartney 1976; according to the most recent tabulation by Penndorf 1957, $p_n = 0.022$ for H_2 and 0.025 for He). If we assume that H is constant in the atmosphere above R, we can write

$$\tau_R = n \frac{32\pi^3 \nu_{STP}^2}{3\lambda^4 L^2} \left[\frac{6 + 3p_n}{6 - 7p_n}\right] (2\pi R H)^{1/2} . \tag{2}$$

Assuming the H_2–He composition used earlier ($H = 50$ km and $\lambda = 5500$ Å) we find that $\tau_R = 1.0$ for the number-density level $n = 4.1 \times 10^{18}$. This level is about 2 scale heights above the top of the CH_4 haze in the model of Weidenschilling and Lewis (1973), and Rayleigh scattering may well define the visible disk. This conclusion should be considered only tentative, since present cloud models for the Uranian upper atmosphere are somewhat uncertain (Lewis, private communication).

To find the altitude difference between the occultation level and the visible disk, we can either appeal to an atmospheric model that encompasses these two levels or extrapolate downward from the occultation level (a distance of 10.6 scale heights in number density). We have chosen the latter method, because the altitude difference we need is not readily obtainable from present models of the upper atmosphere. If we use a mean scale height of 50 km and attach an uncertainty of ± 20 km over the extrapolated range, we find the equatorial radius of the visible disk to be 25,700 $\pm$ 200 km. The ellipticity has been extrapolated to this level also, with the value of J_2 given by Nicholson *et al.* (1978). A comparison of our values with previous determinations of the radius and ellipticity of Uranus is given in Table 3. Apparently there is a real discrepancy between our value for the ellipticity and that obtained from the Stratoscope II images. All other results appear to be consistent within their stated errors.

V. ROTATION PERIOD

From our measured value of the ellipticity and the coefficient for the second zonal harmonic of the Uranian potential field, J_2 (derived from the precession of the ϵ ring), we can obtain the rotation period T for Uranus. For a rotating planet in hydrostatic equilibrium, to first order in J_2 the equation for T is

$$T = 2\pi \left\{\frac{R_e^3(1 - \epsilon)}{2GM[\epsilon - (3/2)(J_2 R_r^2/R_e^2)]}\right\}^{1/2} , \tag{3}$$

where M is the mass of Uranus and R_r is the reference radius for the zonal harmonic expansion (de Sitter 1938; Brouwer and Clemence 1961; Aksnes 1977). We have introduced the reference radius R_r into equation (3) in order to avoid the need for recalculating J_2 for a different choice of R_e. Using our value for $\epsilon = 0.033 \pm 0.007$ and $J_2 = 3.43(\pm 0.02) \times 10^{-3}$ with $R_r = 26{,}200$ km (Nicholson *et al.* 1978), we obtain $T = 12.8 \pm 1.7$ hours. Because of the relative magnitudes of the terms in the denominator of equation (3), the error in T is completely dominated by the error in ϵ.

Our result can be compared to the period of 16.6 $\pm$ 0.5 hr obtained by Franklin *et al.* (1980) from their measurement of the ellipticity and the following rotation periods obtained spectroscopically: 10.8 hr (Lowell 1912; Slipher 1912); 10.8 $\pm$ 0.2 hr (Moore and Menzel 1930); 13.0 $\pm$ 1.3 hr (Trauger, Roesler, and Münch 1978); 24 $\pm$ 3 hr (Hayes and Belton 1977); and 15.57 $\pm$ 0.80 hr (Brown and Goody 1977).

TABLE 3

MEASURED RADII AND ELLIPTICITIES FOR URANUS

Level in the Atmosphere	Equatorial Radius (km)	Ellipticity $(1 - R_p/R_e)$	Comments
Occultation: 1.0×10^{14} molecules cm^{-3}....	26,228 $\pm$ 30	0.033 $\pm$ 0.007	Present work
Unity optical depth at the limb for Rayleigh scattering (visible disk?)..............	25,700 $\pm$ 200	0.035 $\pm$ 0.008	Present work; extrapolated from the occultation level
Visible disk............................	25,400 $\pm$ 280	0.030 $\pm$ 0.008	Double-image micrometer[a]
Visible disk............................	25,900 $\pm$ 300	0.01 $\pm$ 0.01	Measured from stratoscope images[b]
Visible disk............................	...	0.022 $\pm$ 0.001	Obtained from a reanalysis of stratoscope images[c]
Visible disk............................	25,700 $\pm$ 500	...	Lunar occultation[d]

REFERENCES.—[a] Dollfus 1970. [b] Danielson *et al.* 1972. [c] Franklin *et al.* 1980. [d] Radick and Tetley 1980.

VI. CONCLUSIONS

The discrepancies between the various determinations of the ellipticity and rotation rate of Uranus present a major dilemma. Until there is some consensus among spectroscopists about the correct rotation rate, we shall not know whether there is a significant disagreement between the actual rotation rate of the planet and that implied by our value for the ellipticity. In terms of models for the Uranian interior, the ellipticity obtained here, on the basis of models by Podolak and Cameron (1974) and Podolak (1976), would indicate a smaller fraction of volatiles than implied by the longer spectroscopic periods (Goody and Brown 1978). In fact, our result agrees well with the three-layer model of Hubbard and MacFarlane (1980).

The most significant departure of our results from others is the difference in ellipticity from that obtained from the Stratoscope II images. We see two possible explanations. One is that the ellipticity obtained from the Stratoscope II images was affected by variable brightness of Uranus around its limb and does not refer to a surface of constant number density in the atmosphere. Limb and polar brightening have been observed for Uranus in the methane bands (Sinton 1972; Franz and Price 1977; Price and Franz 1978), and small effects at the wavelengths of the Stratoscope II images would not appear unreasonable.

The alternative explanation for the discrepancy is that Uranus is not in hydrostatic equilibrium. In that case, one or both of the ellipticities would not be related to the period as given by equation (3), which requires that R_e and ϵ be obtained for a surface of constant number density (or pressure) for a rotating planet in hydrostatic equilibrium. For Jupiter, the occultation and optical ellipticities showed good agreement (Hubbard and Van Flandern 1972), and the assumption of hydrostatic equilibrium proved to be a good approximation (Anderson 1976).

The precision of the occultation values for both the radius and the ellipticity should considerably improve when further observations can be used to establish the center of the ring system more reliably, and, thus, reduce the main source of uncertainty in our present results.

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