ICARUS 108, 219–224 (1994)

A New Determination of Radii and Limb Parameters for Pluto and Charon from Mutual Event Lightcurves

ELIOT F. YOUNG

NASA Ames Research Center, MS 245-6, Moffett Field, California 94035
E-mail: efy@ssal.arc.nasa.gov

AND

RICHARD P. BINZEL

Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Received August 16, 1993, revised December 21, 1993

Over the past several years Pluto–Charon mutual events have yielded progressively more accurate estimates of Charon's orbital elements and the radii of Pluto and Charon (e.g., Buie, Tholen, and Horne, 1992, *Icarus* 97, 211–227). Analysis of the 1988 stellar occultation by Pluto indicates a radius for Pluto that is about 4%, or 50 km, larger than the mutual event radius of 1151 km. One possible explanation for the discrepancy is that the mutual event modeling treats Pluto and Charon as uniformly bright disks. If they are limb-darkened, the mutual event fits could underestimate their radii. In this paper we use an independent mutual event data set (Young and Binzel, 1992, *Icarus* 102, 134–149) to fit for Pluto and Charon's radii in a manner independent of either object's limb profile or albedo distribution. Our least-squares solution indicates that Pluto's radius is 1164 ± 22.9 km and Charon's radius is 621 ± 20.6 km.

INTRODUCTION

The duration of a mutual event transit or occultation depends on the fraction of the satellite's orbital period over which some overlap exists between Pluto and Charon. This observable quantity (the event duration) depends on the *ratio* of the Pluto and Charon radii over the Pluto–Charon orbital separation. Since the separation is not well known, we solve for R_P/a and R_C/a, the ratios of Pluto's and Charon's radii over Charon's semimajor axis, instead of R_P and R_C.

Ordinarily the radii and limb parameters are too highly correlated to be determined separately. A uniformly bright object might produce the same eclipse lightcurve as a slightly larger, limb-darkened object. Fortunately the mutual events allow us to separate these parameters. In general, the front body casts a shadow that is independent of its limb parameter. If the event lightcurves are divided into two sets, Pluto's radius can be determined from the superior events (Pluto in front) and Charon's from the inferior ones. After finding the radii we can get the limb darkening coefficients from the complementary set of events. Note that these limb parameters are the combined result of several factors (e.g., the albedo distributions of either object). For this reason we will henceforth refer to the Minnaert coefficients as the *effective* limb parameters, since they should not be used to infer scattering properties of either surface.

Our fitting procedure is as follows: we divide the mutual event data set into superior and inferior events. Using only the superior events, we fit for Pluto's radius. Charon's radius and effective limb parameter will be free parameters in the superior-event fit, but we ignore their solutions because of the high correlation coefficients (greater than 0.92) between the rear object's parameters. We obtain Charon's radius independently from the inferior events. Since Pluto is never transited during the superior events, it does not matter whether Pluto's surface is limb darkened, striped, or polka-dotted. We lock Pluto's limb parameter to $k_P = 0.5$, but the fit solutions are completely independent of the front object's limb parameter.

We repeat the procedure for Charon using only inferior events. We lock Charon's limb parameter to $k_C = 0.5$ and fit for Charon's radius and Pluto's radius and effective limb parameter. Once we have determined the Pluto–Charon radii, we can get the limb parameters from the complementary sets of events. We fix the Pluto–Charon radii to their newly determined values and fit for Pluto's limb parameter from the inferior events and Charon's from the superior events. Note that the solutions in this paper supersede the radii listed in Young (1992), which

Presented at the conference "Pluto and Charon," Flagstaff, AZ, July 1993.

0019-1035/94 $6.00

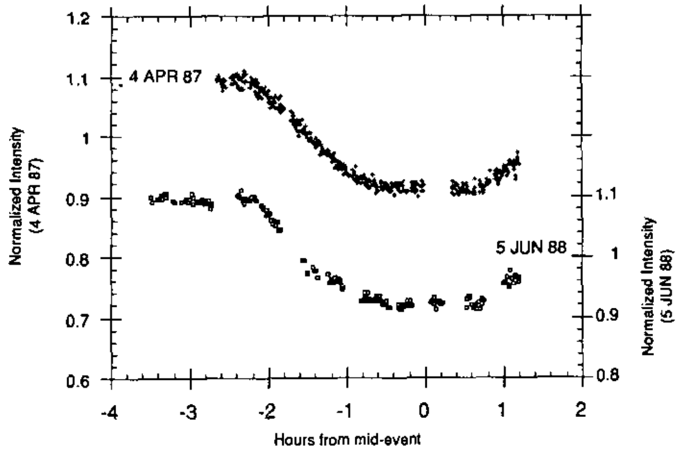

FIG. 1. Composite plot of two superior lightcurves (Pluto in front) taken on 1987 April 4 and 1988 June 5 using a Johnson B filter at the University of Texas McDonald Observatory. The data are in units of Pluto's pre-inferior-event brightness. To get the pre-superior-event baseline, we model Pluto's rotation lightcurve with a polynomial and assume Charon contributes a constant flux. The total number of points is 664.

did not separate mutual event lightcurves into superior and inferior subsets.

DATA

The six inferior event lightcurves were collected at the University of Texas McDonald Observatory as described in Young and Binzel (1992). In this paper we add two superior events that were collected by Binzel at McDonald Observatory using identical instrumentation. These are shown in Fig. 1.

Throughout this paper we assume a Pluto–Charon separation of 19,405 ± 86 km (Null *et al.*, 1993). This distance is used to transform two of the fitted parameters, R_P/a and R_C/a, into radii.

CHARON'S RELATIVE FLUX

Charon's fractional contribution to the lightcurve is a critical input parameter. We assume that Charon's B lightcurve amplitude is zero (Olkin *et al.* 1993), and assess its brightness relative to Pluto from superior events in which Charon is completely covered during midevent. We have made a preliminary four-quadrant model of Charon's albedo distribution based on the five-point B lightcurve of Olkin *et al.* This model indicates that the sub-Pluto and anti-Pluto faces of Charon differ by less than one % in the Johnson B filter.

Pluto's large lightcurve amplitude must be taken into account. We fit a ninth order polynomial to the rotational lightcurve published by Binzel and Mulholland (1983). This fit is shown in Fig. 2.

The best-fit polynomial (shown in Fig. 2) is

$$B(\alpha) = 16.122 - 0.275\,\alpha - 4.999\,\alpha^2 + 18.671\,\alpha^3 + 9.675\,\alpha^4 - 170.42\,\alpha^5 + 356.86\,\alpha^6 - 321.09\,\alpha^7 + 129.98\,\alpha^8 - 18.38\,\alpha^9. \quad (1)$$

Note that the argument of this polynomial is the rotational phase, historically measured from the minimum in the lightcurve (so that midinferior events, for example, occur at a phase of $\alpha = 0.25$). The units are B magnitudes normalized to Pluto at a heliocentric distance of 39.5 AU, a geocentric distance of 38.5 AU, and a solar phase angle of 1.0 degrees. A more complete expression for Pluto's magnitude is given by Tholen and Tedesco (*Icarus*, this issue), consisting of a Fourier series fit to the rotational lightcurve.

We next define the following variables:

P_{pie} = Pluto's pre-inferior-event brightness.

C = Charon's brightness.

f = ratio of Pluto-only flux (measured in mid-event in 1987 and 1988) to Pluto + Charon flux (from the rotational lightcurve of Binzel and Mulholland (1984)).

r_1 = pre-superior-event flux from the Pluto/Charon rotational lightcurve normalized to the system's pre-inferior-event brightness.

r_2 = mid-superior-event flux from the Pluto/Charon rotational lightcurve normalized to the system's pre-inferior-event brightness.

The flux ratios r_1, and r_2 are computed from the polynomial fit to Pluto's rotational lightcurve. The flux ratio f is determined by comparing the flux from events in which Charon is obscured to the mid-event flux from earlier events where there is no obscuration.

If we solve for C in terms of P_{pie} we get

$$C = \frac{x}{1-x} P_{pie}, \quad \text{where } x = r_2 - r_1 \cdot f. \quad (2)$$

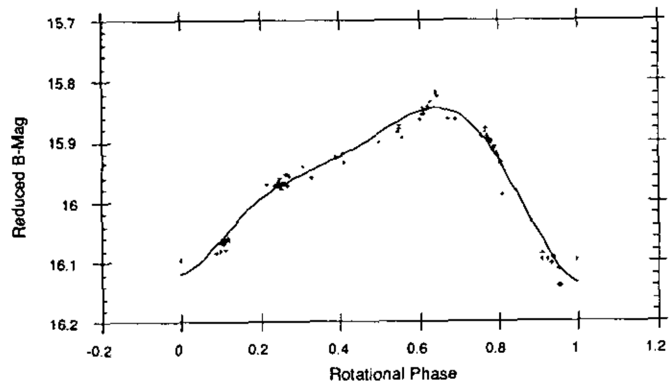

FIG. 2. B-filter rotational lightcurve of Pluto and Charon (Binzel and Mulholland, 1984). The polynomial fit is described in the text.

TABLE I
Superior Event Observations and Charon's Relative Flux

Event	Observer	ΔMag	f	Charon's fractional flux
1987 Apr 04	RPB	−0.196	0.8348	0.2060
1988 Jun 05	RPB	−0.183	0.8449	0.1905

Note: Weighted Average: 0.1982 ± 0.01.

From the two superior lightcurves we find that C is equal to 19.05% or 20.60% of Pluto's pre-inferior-event brightness. The weighted average is 19.82 ± 1.0%. The midevent fluxes from which Charon's fractional flux is derived are shown in Table I.

FITTING PROCEDURE

Pluto and Charon's flux contributions are modeled as the sum of concentric rings. At each moment in time we calculate the exposed, projected area of each ring. The chosen resolution is 100 rings per disk. The modeled flux is the sum of all the exposed rings, each weighted by its effective Minnaert brightness and its exposed fractional area. The central brightness, I_{ctr} is not known *a priori* but is a function of the average surface brightness and the limb parameter, k:

$$I_{ctr} = I_{ave}(k + 0.5). \quad (3)$$

Note that if $k = 0.5$, the entire disk has a uniform surface brightness (no limb effect). The moon is an example of an object with virtually no limb effect. Tholen and Buie (Buie *et al.* 1992) implicitly use a limb coefficient of 0.5 in their orbit solution and radii determination. An object with a limb parameter of 1 is a Lambertian surface (reflecting isotropically with no absorption). There are no known objects in the solar system with limb coefficients greater than 0.8.

Since Pluto's phase angle is so small, we assume that the angle of illumination and observation are coincident, leading to the simplified form of the Minnaert limb darkening rule

$$I(\theta) = I_{ctr}(\cos\theta)^{2k-1}, \quad (4)$$

where θ is the surface normal with respect to the solar incident rays and k is the Minnaert coefficient. The circularly symmetric model is illustrated in Fig. 3.

The Minnaert law is an empirical one and not without its shortcomings. As Fig. 4 illustrates, the limb profile suddenly tends to zero as θ approaches ±90°, except for the special case of $k = 0.5$. This part of the limb profile is especially important when comparing the model lightcurves to grazing event (1990 events) observations. Bear in mind, however, that this paper's radii determinations are *independent* of any limb profile or surface albedo distribution.

The nonlinear least-squares fit was implemented with Marquardt–Levenberg and simplex algorithms outlined in *Numerical Recipes in C* (Press *et al.*, 1992). Each single iteration used a singular value decomposition (SVD) scheme to avoid potential problems with highly correlated unknowns. We found that the chi-squared "valley" was extremely flat near the solution and had to be explored by the slow but steady simplex algorithm. The formal errors of the fit were estimated by running the SVD routine for an iteration at the simplex algorithm's solution.

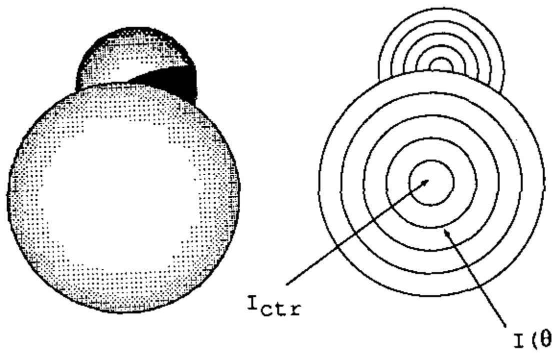

FIG. 3. Gray-scale diagram of the Minnaert model of Pluto and Charon generated from arbitrary values of $k_P = 0.6$ and $k_C = 0.7$. Note that the models are circularly symmetric and can be represented as sums of concentric rings.

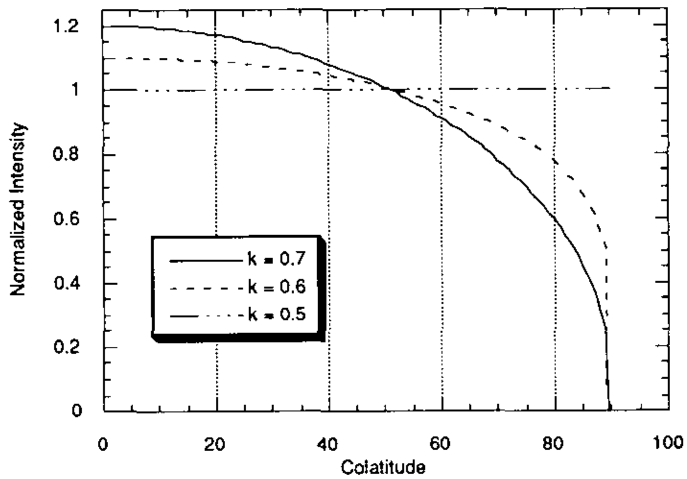

FIG. 4. Normalized intensity versus angle from the subsolar point for the Minnaert law, depicting the steep effect as θ approaches 90°. The profile is illustrated for three values of the parameter k.

TABLE II
Solution for R_P/a

Parameter	Value
k_P	0.50 (fixed)
R_P/a	0.0599 ± 0.00029
R_C/a	0.0322[a]
k_C	1.07[a]
χ^2	8.74

[a] Correlation coefficient for R_C/a and k_C = 0.94.

FIT RESULTS

Superior lightcurves (Pluto in front) are used to fit for Pluto's radius, since the dimensions of the front object are relatively insensitive to the interaction between the rear object's radius and limb parameter. Why is this so? Regardless of Charon's degree of effective limb darkening, the apparent onsets of first and third contact (or second and fourth contact) are determined by nearly identical geometries. Thus the *time interval* between first and third contact is well known and this time interval is an excellent constraint on the radius of the front object. Tables II and III summarize the independent fits for the radii of Pluto and Charon, respectively. Although Table II presents a radius for Charon, we discard this value in favor of the inferior-event solution shown in Table III. Using the newly determined radii we can get each object's effective limb parameter from the complementary set of events. The effective limb coefficient solutions are shown in Tables IV and V. Pluto's effective limb parameter falls out of the inferior event set when R_p, R_c, and k_c are fixed. The best fit for Pluto's effective limb parameter is k_p = 0.49 ± 0.017, basically indistinguishable from the non-limb-darkened value 0.5. The best fit for Charon's effective limb parameter is k_c = 1.02 ± 0.06.

A semimajor axis of 19405 ± 86 km (Null *et al.* 1993) yields radii of 1164 ± 21.5 km and 624 ± 15.7 km for Pluto and Charon respectively. It is interesting to note that a milestone of sorts has been reached; the semimajor axis's uncertainty (previously 320 km) is no longer the dominant source of error in the radii determinations. In propagating the errors it is important to include the uncertainty in Charon's fractional flux (0.1982 ± 0.01). This error source overshadows the formal errors from the least-squares fit.[1]

Charon's orbital elements are another possible error source. The recent results of Null *et al.* (1993) and Young *et al.* (1993) make significant changes in Charon's semimajor axis and inclination. The eccentricity is zero for our purposes and the other three angles are well determined from the mutual events. Since the inclinations of Null and Young are 2–3 percent smaller than that of Tholen and Buie (1989), we tested our solution's sensitivity to the inclination. We found that the partial derivatives of R_P and R_C with respect to Charon's inclination are −3.9 and −5.2 km/°. With an assumed error of 2.5° in the inclination (based on the discrepancy between the mutual event result (Tholen and Buie, 1989) and the more recent astrometry (Null *et al.* 1993)), we find that Pluto's radius solution is affected minimally by the uncertainty in the inclination. Charon's radius is significantly affected, however, since

TABLE III
Solution for R_C/a

Parameter	Value
k_C	0.50 (fixed)
R_C/a	0.0320 ± 0.00079
R_P/a	0.0567[a]
k_P	0.30[a]
χ^2	17.9

[a] Correlation coefficient for R_P/a and k_C = 0.92.

TABLE IV
Solution for k_P

Parameter	Value
k_C	0.50 (fixed)
R_P/a	0.0599 ± 0.00029 (fixed)
R_C/a	0.0320 ± 0.00079 (fixed)
k_P	0.49 ± 0.017
χ^2	17.9

TABLE V
Solution for k_C

Parameter	Value
k_P	0.50 (fixed)
R_P/a	0.0599 ± 0.00029 (fixed)
R_C/a	0.0320 ± 0.00079 (fixed)
k_C	1.02 ± 0.06
χ^2	8.74

[1] How sensitive are the radii solutions to the assumed value of Charon's flux? The partial derivatives of R_P/a and R_C/a with respect to C were determined numerically and found to be 0.08 and 0.017, respectively. These partial derivatives are used in the propagation of errors. The magnitude of the error terms related to the uncertainty in Charon's fractional flux outweigh the internal errors of the fit by roughly a factor of 5.

TABLE VI
Comparison with Mutual Event Solutions
(k_P and k_C Locked to 0.5)

	R_P	R_C	k_P	k_C
Sup	0.0601	0.0286	0.5	0.5
Inf	0.0598	0.0321	0.5	0.5
Both	0.0601	0.0320	0.5	0.5

±13 km (the error due to the uncertainty in the inclination) is an appreciable fraction of the other errors, ±15.7 km. Charon's total error is therefore ±20.6 km, and Pluto's increases slightly to ±22.9 km. The errors in Pluto's and Charon's effective limb parameters increase from 0.017 and 0.06 to 0.019 and 0.078 respectively.

We performed the fit for Pluto's and Charon's radii with both effective limb parameters fixed to 0.5 to compare our results with Tholen and Buie's results. We found that Pluto's radius is consistent with the Tholen and Buie value, which is not surprising given that Pluto's effective limb solution was very close to 0.5 anyway. Charon, on the other hand, was found to have a radius of 621 km based on the inferior events and only 555 km based on the superior events, spanning the Tholen and Buie value of 593 km. These results are shown in Table VI. Recall that our solution for Charon's radius is closer to the inferior event value of 621 km because the inferior event fits are robust against variations in Charon's limb darkening or albedo distribution.

TESTING THE SOLUTION

A valuable test of this solution is the comparison of selected 1990 event lightcurves to lightcurves generated from the model. Observations of these grazing events seem to indicate that no significant overlap took place (Tholen and Tedesco 1994), thereby providing a strong upper limit on Pluto's and Charon's radii. Tholen observed "ruler-flat"[2] lightcurves on April 16 and May 18 (inferior) and on March 31 and May 2 (superior). We have generated model lightcurves of these events (shown in Figs. 5 and 6).

The nominal solution yields superior and inferior lightcurves with depths of 0.007 and 0.004 magnitudes respectively. This event depth should be detectable in Tholen's lightcurves. The one-sigma envelope reduces the event depth to 0.004 and 0.002 magnitudes for the superior and inferior events, which is marginally consistent with Tholen's 1990 lightcurves. The depth of the events is not due to Pluto's radius, which is not significantly larger than Tholen and Buie's radius (1989), but due to Charon's large radius and the decreased separation between the two objects (Null *et al.* 1993). An obvious project for future study is a radius solution that includes these grazing lightcurves.

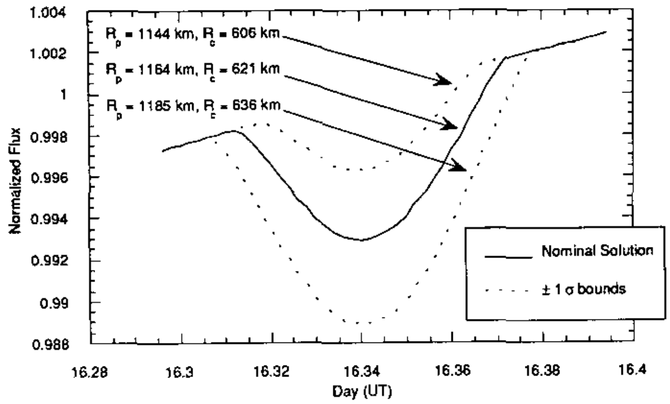

FIG. 5. Inferior event model lightcurves for April 16, 1990, showing the predicted event depths for a variety of model parameters.

Figure 7 shows the importance of modeling the limb profile. This figure shows lightcurves that were generated with two different models of Pluto: one which has no limb darkening ($k = 0.5$) and a radius of 1164 km, and another with a limb parameter of 0.7 and a radius of 1193 km. These two lightcurves are nearly identical. We conclude that the limb effect or surface albedo distribution must be taken into account if the mutual events are to yield radii within 5% accuracy.

SUMMARY

The fit results yield Pluto and Charon radii of 1164 ± 22.9 km and 621 ± 20.6 km and effective limb coefficients

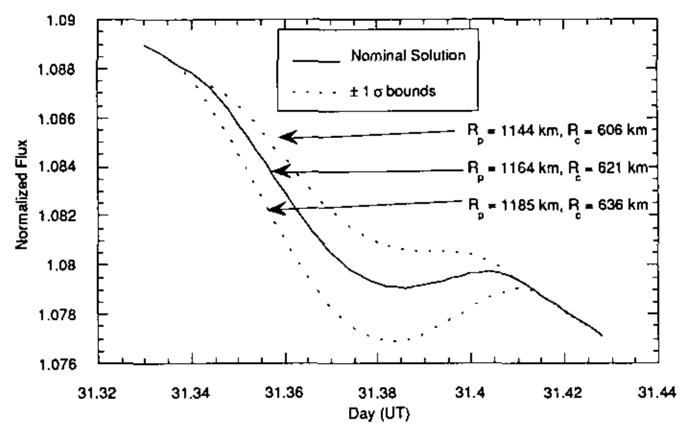

FIG. 6. Superior event model lightcurves for March 31, 1990, showing the predicted event depths for a variety of model parameters.

[2] "Ruler flat" in this case means that within the rms error of 0.0025 magnitudes no dip in the lightcurve could be discerned (Tholen, personal communication).

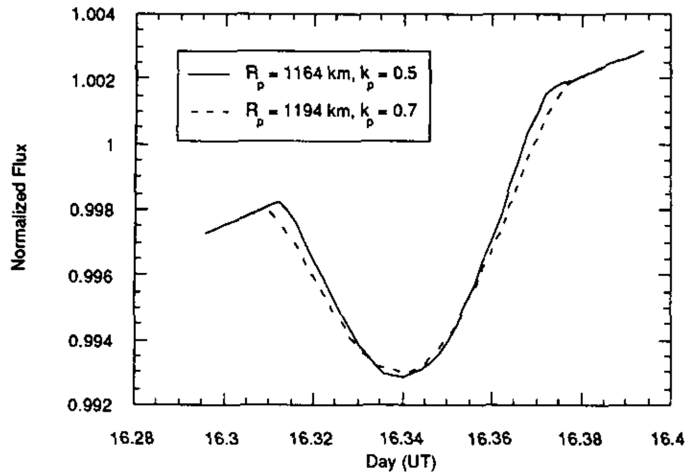

FIG. 7. Illustration of model lightcurves for April 16, 1990, showing similar event depths for different values of the limb darkening parameter and radii.

of $k_P = 0.49 \pm 0.019$ and $k_C = 1.02 \pm 0.078$. We stress that these limb parameters should not be used to infer surface scattering properties. The Pluto radius of 1164 km is compatible with the previous mutual event work but is still lower than the stellar occultation thermal-gradient value of 1195 ± 5 km (Millis *et al.* 1993). The haze model radius of Millis *et al.* has an upper limit at $R_P < 1180 \pm$ km; this model is compatible with our value of R_P. Another explanation of the stellar occultation's large radius solution, proposed by Stansberry *et al.* (1993), is that Pluto has a troposphere which, like a haze layer, would not be probed by the occultation.

While Pluto's radius is consistent with the previous mutual event radius of 1151 ± 7 km, Charon's radius is not. The fitted value of 621 ± 20.6 km is larger than the 593 ± 10 km value of Buie *et al.* (1992). This larger radius is consistent with the 601 km lower limit found for Charon's radius through an analysis of a 1980 stellar occultation (Elliot and Young 1991).

We find that Pluto has virtually no effective limb darkening. Charon's best-fit limb parameter is surprisingly large, however, at 1.02 ± 0.078. Note that Charon casts a large shadow in the inferior event set but appears smaller when it is obscured by Pluto during the superior ones; our model accommodates this by attributing a large effective limb coefficient to Charon. The cause of this large effective limb parameter may be the albedo distribution of Charon's sub-Pluto hemisphere.

We believe that dividing the event set into superior and inferior lightcurves has proved to be a simple way to separate radii from limb parameters. We advocate this procedure in future fitting work.

REFERENCES

Beletic, J. W., R. M. Goody, and D. J. Tholen 1989. Orbital elements of Charon from speckle interferometry. *Icarus* **79**, 38–46.

Binzel, R. P., and J. D. Mulholland 1984. Photometry of Pluto during the 1983 opposition: A new determination of the phase coefficient. *Astron. J.* **89**, 1759–1761.

Bosh, A. S., L. A. Young, J. L. Elliot, H. B. Hammel, and R. L. Baron, 1992. Photometric variability of Charon at 2.2 μm. *Icarus* **95**, 319–324.

Buie, M. W., D. J. Tholen, and K. Horne 1992. The surface albedo distribution of Pluto. *Icarus* **97**, 211–227.

Elliot, J. L., E. W. Dunham, A. S. Bosh, S. M. Slivan, L. A. Young, L. H. Wasserman, and R. L. Millis 1989. Pluto's atmosphere. *Icarus* **77**, 148–170.

Elliot J. L. and L. A. Young 1991. Limits on the radius and a possible atmosphere of Charon from its 1980 stellar occultation. *Icarus* **89**, 244–254.

Elliot, J. L., and L. A. Young 1992. Analysis of stellar occultation data for planetary atmospheres. I. Model fitting with application to Pluto. *Astron. J.* **103**, 990–1015.

Millis, R. L., L. H. Wasserman, O. G. Franz, R. A. Nye, J. L. Elliot, E. W. Dunham, A. J. Bosh, L. A. Young, S. W. Slivan, A. C. Gilmore, P. M. Kilmartin, W. H. Allen, R. D. Watson, S. W. Dieters, K. M. Hill, A. B. Giles, G. Blow, J. Priestly, W. M. Kissling, W. S. G. Walker, B. F. Marino, D. G. Dix, A. Page, J. E. Ross, M. D. Kennedy, K. A. Mottram, G. Mayland, T. Murphy, C. C. Dohn, and A. R. Klemola 1993. Pluto's radius and atmosphere: Results from the entire 9 June 1988 occultation data set. *Icarus* **105**, 282–297.

Null, G. W., W. M. Owen, and S. P. Synnott, 1993. Masses and densities of Pluto and Charon. *Astron. J.* **105**, 2319–2335.

Olkin, C. B., L. A. Young, J. L. Elliot, D. J. Tholen, and M. W. Buie 1993. Individual lightcurves of Pluto and Charon. Presented at the Pluto–Charon Meeting, Flagstaff, AZ, 1993.

Press, W. H., B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling 1988. *Numerical Recipes in C.* Press Syndicate of the University of Cambridge.

Stansberry, J. A., J. I. Lunine, W. B. Hubbard, R. V. Yelle, and D. M. Hunten, 1993. Mirages and the nature of Pluto's atmosphere. Submitted for publication.

Tholen, D. J., and E. F. Tedesco 1994. Pluto's lightcurve: Results from four oppositions. *Icarus* **108**, 200–208.

Tholen, D. J., and M. W. Buie 1989. Circumstances for Pluto–Charon mutual events in 1990. *Astron. J.* **96**, 1977–1982.

Tholen, D. J., and M. W. Buie, 1991. How big is Pluto? *Bull. Am. Astron. Soc.* **23**, 1216.

Veverka, J., P. Thomas, T. V. Johnson, and D. Matson, 1986. The physical characteristics of satellite surfaces. In *Satellites* (J. Burns and M. S. Matthews, Eds.), Chap. 8. Univ. of Arizona Press, Tucson, AZ.

Young, E. F. 1992. *An Albedo Map and Frost Model of Pluto.* Sc.D. Thesis, Massachusetts Institute of Technology.

Young, E. F., and R. P. Binzel 1993. Comparative mapping of Pluto's sub-Charon hemisphere: Three least squares models based on mutual event lightcurves. *Icarus* **102**, 134–149.

Young, L. A., C. B. Olkin, J. L. Elliot, D. J. Tholen, and M. W. Buie 1993. Evidence for a dense Charon from MKO astrometry. *Bull. Am. Astron. Soc.* **25**, 1137.