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OCEAN PLANET OR THICK ATMOSPHERE: ON THE MASS-RADIUS RELATIONSHIP FOR SOLID EXOPLANETS WITH MASSIVE ATMOSPHERES

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ABSTRACT

The bulk composition of an exoplanet is commonly inferred from its average density. For small planets, however, the average density is not unique within the range of compositions. Variations of a number of important planetary parameters—which are difficult or impossible to constrain from measurements alone—produce planets with the same average densities but widely varying bulk compositions. We find that adding a gas envelope equivalent to 0.1%–10% of the mass of a solid planet causes the radius to increase 5%–60% above its gas-free value. A planet with a given mass and radius might have substantial water ice content (a so-called ocean planet), or alternatively a large rocky iron core and some H and/or He. For example, a wide variety of compositions can explain the observed radius of GJ 436b, although all models require some H/He. We conclude that the identification of water worlds based on the mass-radius relationship alone is impossible unless a significant gas layer can be ruled out by other means.

Subject headings: planets and satellites: general — planetary systems — stars: individual (GJ 436)

Online material: color figures

1. INTRODUCTION

Out of over 250 exoplanets known to date, over 20 are known to transit their stars. Transiting planets are important because we can derive the precise mass and radius, and can begin to determine other planetary properties, such as the bulk composition.

Much attention has been given to "ocean planets" or "water worlds," planets composed mostly of solid water (Kuchner 2003; Léger et al. 2004). If a water world is found close to a star, it would be strong evidence for migration, because insufficient volatiles exist near the star for in situ formation. The proposed identification of water worlds is through transits. From a measured mass and radius, a low-density water planet could potentially be identified.

We examine the possibility that water worlds cannot be uniquely identified based on the mass and radius of a transiting planet. An alternative interpretation could be a rocky planet with a thick hydrogen-rich atmosphere. Most authors have assumed that solid planets in the 5–10 $M_\oplus$ range have an insignificant amount of hydrogen (Valencia et al. 2006, 2007; Fortney et al. 2007; Seager et al. 2007; Sotin et al. 2007), with the exception of Selsis et al. (2007), who have qualitatively discussed the degeneracy between ocean worlds and planets with H_2 atmospheres.

Exoplanets have, however, contradicted our basic assumptions before. Notable examples include the existence of hot Jupiters; the predominance of giant planets in eccentric orbits; and the gas-rock hybrid planet HD 149026b, with its $\sim$60 $M_\oplus$ core and $\sim$30 $M_\oplus$ H/He envelope (Sato et al. 2005).

We adopt the idea that a wide range of atmospheric formation and loss mechanisms exist and can lead to a range of atmosphere masses on different exoplanets. We explore the mass-radius relationship for the lowest mass exoplanets yet detected ($\sim$5–20 $M_\oplus$) in order to identify potential ambiguities that result from the presence of a massive atmosphere. We explore atmospheres ranging from $\sim 10^{-3}$ $M_\oplus$ (10 times Venus' atmospheric mass) to $\sim$1 $M_\oplus$ (the estimated mass of Uranus' and Neptune's H/He; Guillot 2005; Podolak et al. 1995; Hubbard et al. 1991), with a focus on the smaller mass range. We also explore potential compositions for the transiting Neptune-size planet GJ 436b (Butler et al. 2004; Gillon et al. 2007a).

2. MODELS

Our model assumes a spherical planet in hydrostatic equilibrium, with concentric shells of different composition sorted by density. We solve for the mass $m(r)$, pressure $P(r)$, density $\rho(r)$, and temperature $T(r)$ using the equation for mass of a spherical shell,

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r), \tag{1}$$

the equation of hydrostatic equilibrium,

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\rho(r)}{r^2}, \tag{2}$$

and the equation of state of the material,

$$\rho(r) = f(T(r), P(r)). \tag{3}$$

To build a model planet, the composition and mass of up to three layers are specified. Starting from a central pressure adjusted to achieve the desired mass, the differential equations for $P(r)$ and $m(r)$ are numerically integrated outward, and $\rho(r)$ is calculated from the equation of state for the material. At the boundary between layers, $P(r)$ and $m(r)$ are used as the initial conditions for the next layer. The integration is stopped at the outer boundary condition $P(r) = 1$ bar, approximately where the atmosphere ceases to be opaque at visible and infrared wavelengths.[3]

The temperature of a planet's gas layer has a significant effect on the radius. Instead of computing a cooling history to obtain $T(r)$, we calculate $T(r)$, separately in three different regimes. Within the solid portion of the planet ($r \geq r_{\rm solid}$), temperature has little effect on the final radius, and is assumed to be isothermal (see Seager et al. 2007). In the deep hydrogen-helium layer where the

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[3] We ignore any wavelength-dependent effects on potential measured radii of extrasolar planets.

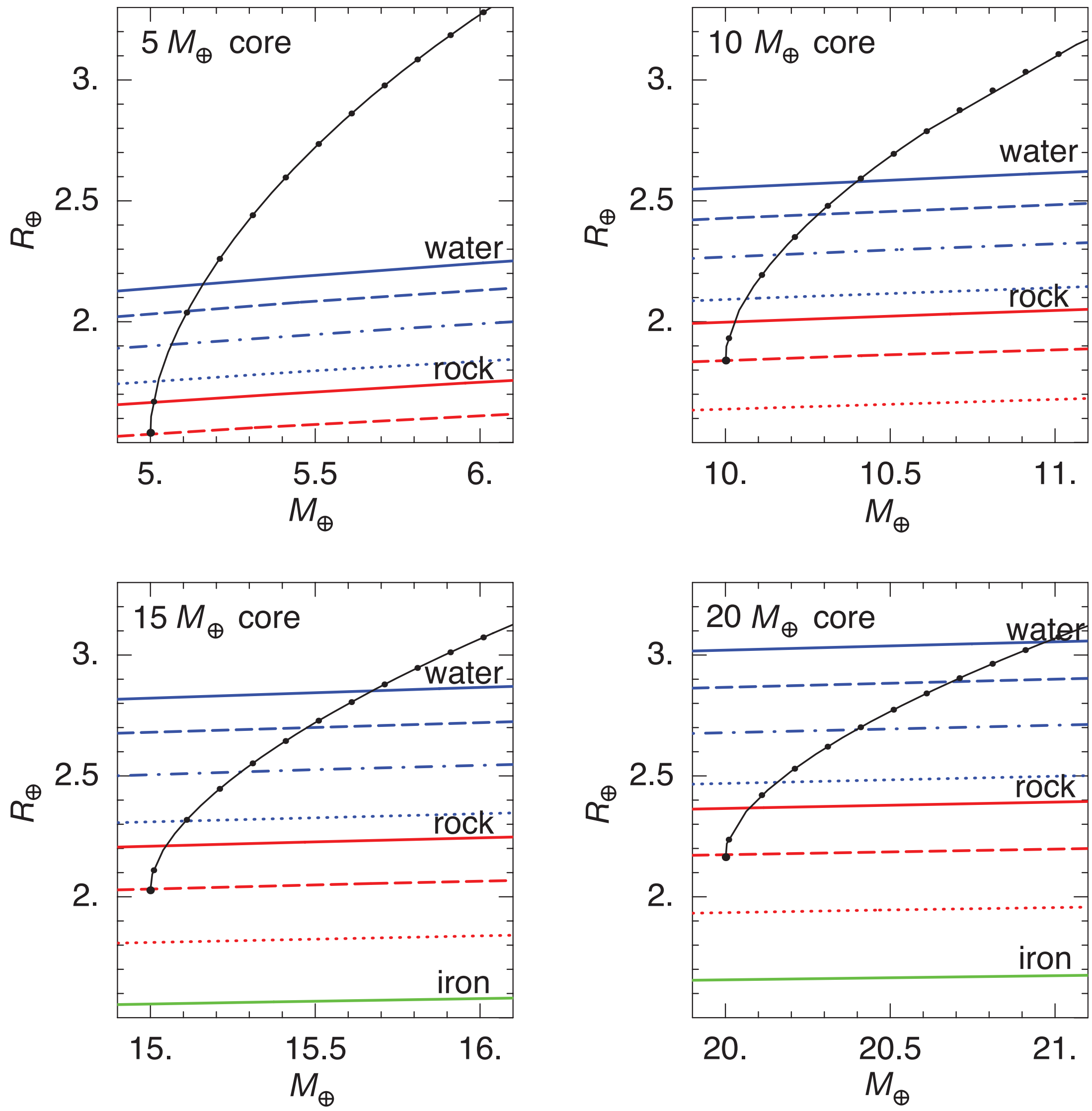

FIG. 1.—Increase in radius due to adding H/He to a solid planet. A H/He layer of 0.002–1 $M_\oplus$ is added to a solid planet of 5, 10, 15, or 20 $M_\oplus$, with fiducial model parameters (30% Fe and 70% $MgSiO_3$). The black points are for atmospheres at 0.01 $M_\oplus$ and every 0.1 $M_\oplus$ afterward. The mass-radius relationship of solid planets with no gas is plotted for comparison. The water (*blue*), rock (*red*), and iron (*green*) curves are taken from Seager et al. (2007) and represent homogeneous solid planets. Intermediate compositions for differentiated planets are, from top down: *dashed blue line*: 75% H_2O, 22% $MgSiO_3$, and 3% Fe; *dashed-dotted blue line*: 48% H_2O, 48.5% $MgSiO_3$, and 6.5% Fe; *dotted blue line*: 25% H_2O, 52.5% $MgSiO_3$, and 22.5% Fe; *dashed red line*: 67.5% $MgSiO_3$ and 32.5% Fe; *dotted red line*: 30% $MgSiO_3$ and 70% Fe. In general, the addition of a gas layer of up to ~5% of the solid planet mass will inflate the radius of a rocky iron planet through the range of radii corresponding to water planets with different water mass fractions.

pressure is greater than 1 kbar ($r_{\rm solid} \leq r \leq r_{1\ \rm kbar}$), we assume that convection dominates and the temperature follows an adiabat tied to the entropy, S, at 1 kbar. At lower pressures ($r > r_{1\ \rm kbar}$), we use the radiative equilibrium gray analytical model of Chevallier et al. (2007) and Hansen (2007) for irradiated atmospheres,

$$T(r) = \frac{3}{4} T_{\rm eff}^4 \left(\tau + \frac{2}{3}\right) + T_{\rm eq}^4 F(\tau, \mu_0, \gamma), \qquad (4)$$

where τ is the optical depth and $\mu_0 = \cos\theta_0$, where θ_0 is the angle of incident radiation with respect to the surface normal, and γ is a parameter that accounts for the altitude at which radiation is absorbed. We convert from optical depth to pressure through a constant scaling relation under the assumption of hydrostatic equilibrium and constant opacity per gram, following Hansen (2007) but taking into account the planet's surface gravity. Here $T_{\rm eff}$ is the effective temperature in the absence of stellar irradiation, representing energy from internal sources, and $T_{\rm eq}$ is the equilibrium temperature, which represents heating from the parent star.

We emphasize that by choosing an effective temperature—or adiabat—we are subsuming a cooling calculation. This is common practice for modeling the interior structure of solar system giant planets (e.g., Stevenson 1982; Hubbard et al. 1991; Marley et al. 1995; Saumon & Guillot 2004) and was also the case even before their $T_{\rm eff}$ and gravitational moments were known or well known (Demarcus 1958; Zapolsky & Salpeter 1969). In fact, even with a known $T_{\rm eff}$, cooling calculations only match observed parameters for Jupiter (e.g., Hubbard 1977); Saturn is hotter than expected (Pollack et al. 1977; Stevenson & Salpeter 1977; Fortney & Hubbard 2003), and Uranus and Neptune are colder than expected (Stevenson 1982; Podolak et al. 1991; Guillot 2005). An additional motivation to adopt a simple framework for temperature is the uncertainty in interior parameters for rocky planets.

This includes the temperature-dependent equation of state of liquids and solids at high temperature and pressure, the temperature-dependent viscosity, and the effect of tides on the cooling history for eccentric exoplanets. We further note that our choice of radiative equilibrium down to 1 kbar is based on the irradiated atmosphere/interior models by Fortney et al. (2007). For our fiducial model, we choose $T_{\rm eff}$ to be similar to Earth's and Uranus's. While the largest uncertainty in our treatment is the qualitative choice of $T_{\rm eff}$, we subsequently vary over a reasonable range of effective temperatures (or adiabats).

We use the H/He equation of state from Saumon et al. (1995), ignoring the "plasma-phase transition," which may be a numerical artifact (D. Saumon 2007, private communication). The equations of state for the solid materials Fe (Anderson et al. 2001), $MgSiO_3$ perovskite (Karki et al. 2000), $(Mg,Fe)SiO_3$ (Knittle & Jeanloz 1987), and H_2O are described in more detail in Seager et al. (2007).

3. RESULTS AND DISCUSSION

Our fiducial planet consists of a 30% Fe core and a 70% $MgSiO_3$ mantle, roughly analogous to Earth. We used a H/He mixture with helium mass fraction $Y = 0.28$ (the He mass fraction of the solar nebula). We chose $T_{\rm eq} = 300$ K, based on the observation that a planet around an M dwarf at an orbital distance of 0.1 AU has a similar equilibrium temperature to Earth (assuming similar albedos). We set $T_{\rm eff} = 30$ K, similar to Earth and Uranus[4]. For the atmospheric parameters, we fixed $\mu_0 = \cos 60°$ and $\gamma = 0.1$ to represent radiation absorbed deep in the atmosphere.[5] We later investigate variations on Y, $T_{\rm eff}$, $T_{\rm eq}$, and γ. Figure 1 shows a plot of the mass-radius relationship for fiducial planets of masses 5, 10, 15, and 20 $M_\oplus$. For each planet mass, we added atmospheres ranging in mass from 0.001–1 $M_\oplus$.

A robust finding for all models is that a small amount of gas creates a large radius increase. While this result is expected, the radius increase is far more dramatic than anticipated.

For example, an H/He atmosphere of $\sim$0.001 by mass—only 10 times greater than Venus' atmospheric mass fraction—is required for a noticeable radius increase. As seen in Figure 1, adding a hydrogen-helium atmosphere with just 0.1% of the mass of a 10 $M_\oplus$ rocky planet results in a 5% increase in the planetary radius—within a measurement precision that has been obtained for currently known transiting planets.

As a second example, adding a gas layer of H/He equal to 1% of the mass of our fiducial planets increases the radius by $\sim$20% of the original planet radius, or by about 0.35 $R_\oplus$ for the planet masses we considered.

Our major finding is that exoplanets with a significant H/He layer cannot be distinguished from water worlds, based on M_p and R_p alone. For our fiducial solid exoplanets, adding up to 5% H/He by mass (for 10 $M_\oplus$ planets) is sufficient to push the planet's radius through the entire range of radii corresponding to solid planets with no gas, including planets with up to 100% water composition. While we have not completed an exhaustive study of possible compositions, we find the nonuniqueness of water planets to be valid for any conditions we investigated.

This generic finding holds for a wide range of assumptions of assumed temperatures. Taking our fiducial model, we vary $T_{\rm eq}$, $T_{\rm eff}$, and γ individually. For a 10 $M_\oplus$ solid planet with an additional 0.1 $M_\oplus$ H/He atmosphere, increasing $T_{\rm eq}$ from 300 to 500 K increases the radius by about 1% (Fig. 2). For the same planet, varying $T_{\rm eff}$ from 10 to 50 K results in an 8% increase in radius (Fig. 3). While large, this value is comparable to expected radius uncertainties for these planets (Gillon et al. 2007b, 2007a; Deming et al. 2007). Varying the altitude where radiation is absorbed (specified by γ) has a much smaller effect on the planet radius. Varying γ from 0.1 to 10 causes the radius to decrease by 0.2%.

A corollary of our main result is that when a planet has a significant H/He atmosphere, there is a wide degeneracy in allowable internal composition. This is not just compositional, but also relates to the trade-off of temperature and mass of H/He gas. It could be argued that specifying a planet's composition implies a particular internal thermal profile derived from a consistent cooling history. As addressed in § 2, the many unknowns and free input parameters for rocky planet interiors—such as the possible differences between atmospheric and interior compositions, equation of states, and the effect of tides on the planet's cooling history—prevent a self-consistent solution at present.

How could a 5–20 $M_\oplus$ exoplanet get a substantial H/He layer? Two different scenarios may produce them: direct capture of gas from the protoplanetary disk (possibly modified by the escape of some fraction of the original gas), or outgassing during accretion.

A planet may capture and retain up to 1–2 $M_\oplus$ of H/He if the planetary core did not grow quickly enough to capture more

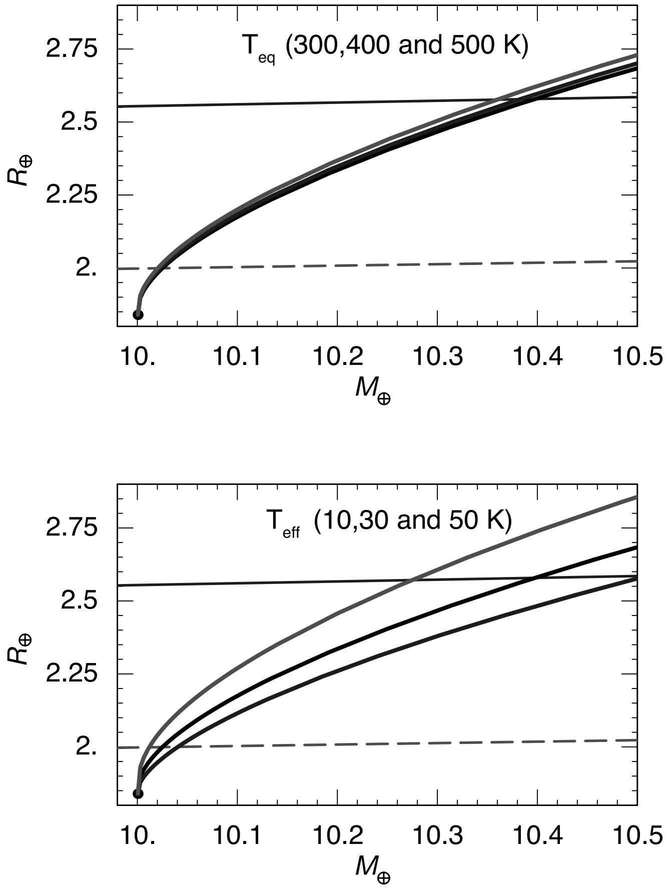

FIG. 2.—Effects on the radius of varying the equilibrium temperature (*top*) and effective temperature (*bottom*) for a 10 $M_\oplus$ planet with otherwise fiducial parameters. $T_{\rm eq}$ values of 300, 400, and 500 K are plotted to simulate the effect of uncertainty in the orbital parameters and albedo on the expected radius. $T_{\rm eff}$ values of 10, 30, and 50 K are plotted on the same scale as the $T_{\rm eq}$ plot, to show uncertainties in the planet's interior temperature. Uncertainties in the internal energy of a planet lead to large variations in radii for a given mass, showing how temperature is a large uncertainty in the interpretation of a planet's internal composition. [*See the electronic edition of the Journal for a color version of this figure.*]

[4] Earth has 44×10^{12} W (Pollack et al. 1993) and Uranus has 340×10^{12} W of energy flow (Pearl et al. 1990).

[5] In comparison, $\gamma = 10$ would correspond to absorption high in the atmosphere.

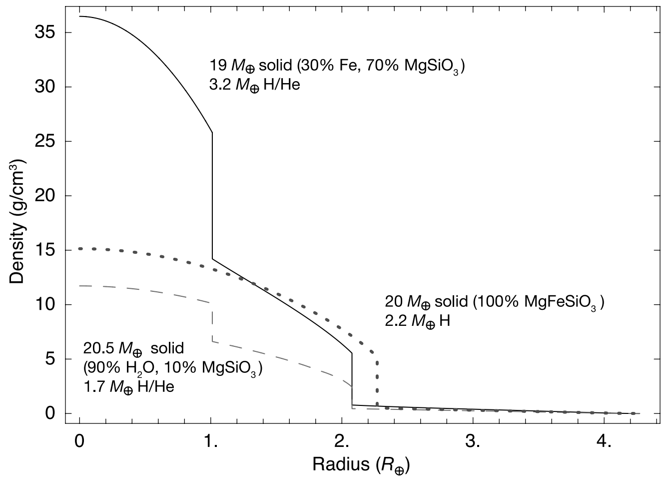

FIG. 3.—Density vs. radius for three different potential compositions of GJ 436b. From top to bottom: *solid curve*, 19.0 $M_\oplus$ core (30% Fe, 70% $MgSiO_3$) with 3.2 $M_\oplus$ H/He ($Y = 0.28$); *dotted curve*, 20.0 $M_\oplus$ core [100% $(Mg,Fe)SiO_3$] with 2.2 $M_\oplus$ H ($Y = 0$); *dashed curve*, 20.5 $M_\oplus$ core (90% H_2O, 10% $MgSiO_3$) with 1.7 $M_\oplus$ H/He ($Y = 0.28$). All three planets have the same total radius (4.3 $R_\oplus$) and total mass (22.2 $M_\oplus$). [*See the electronic edition of the Journal for a color version of this figure.*]

before the gas in the disk evaporated (as is the paradigm for Uranus and Neptune). Alternatively, for short-period exoplanets, a 1–2 $M_\oplus$ H/He envelope may result after substantial loss of an initially massive gas envelope from irradiated evaporation (e.g., Baraffe et al. 2006). Alibert et al. (2006) consider atmospheric evaporation during migration, and conclude that the 10 $M_\oplus$ innermost planet in HD 69830, at 0.08 AU, kept $\sim$2 $M_\oplus$ of H/He over the 4 Gyr lifetime of the star.

Little attention has been given to the mass and composition of exoplanet atmospheres from outgassing. Venus' atmosphere is 10^{-4} $M_\oplus$; if Venus had a surface gravity high enough to prevent H escape, its atmosphere would be over 10^{-3} $M_\oplus$. Even more massive H-rich atmospheres are possible. If a massive iron-silicate planet formed with enough water, the iron may react with the water during differentiation, liberating hydrogen gas (Ringwood 1979; Waenke & Dreibus 1994). L. Elkins-Tanton et al. (submitted) estimate that the maximum H component is about 6% by mass for a terrestrial-composition planet. For a 10 $M_\oplus$ planet, this would result in a 0.6 $M_\oplus$ H envelope.

For short-period, low-mass planets, theoretical arguments of atmospheric escape may be the best way to identify a water world based on the mass and radius measurements alone (Léger et al. 2004; Selsis et al. 2007). Indeed, our assumption of H/He atmospheres for exoplanets relies on the condition that atmospheric mass loss has not evaporated all of the H/He. In the absence of hydrodynamic escape, the exospheric temperature (and not the atmospheric $T_{\rm eff}$) drives the thermal Jeans escape of light gases. Earth and Jupiter both have exobase temperatures of 1000 K (de Pater & Lissauer 2001), significantly above their $T_{\rm eff}$ of 255 and 124.4 K, respectively (Cox 2000). Uranus and Neptune have exobase temperatures around 750 K (de Pater & Lissauer 2001). We note that because GJ 436 must have at least 1 $M_\oplus$ of H/He, its exospheric temperature is not too high. At the other extreme, planets of 5 $M_\oplus$ would require very low exospheric temperatures ($\sim$300 K) to retain a massive atmosphere over the course of billions of years. Nevertheless, a young 5 $M_\oplus$ Earth-mass planet with a captured atmosphere could still have a H/He atmosphere, and an old 5 $M_\oplus$ planet could retain a substantial He fraction, making its compositional identification ambiguous.

We now turn to a qualitative study of GJ 436b, to show that the interpretation that GJ 436b is a water world akin to Uranus and Neptune (Gillon et al. 2007a) is not the only possibility. We consider the GJ 436b values $M_p = 22.2$ $M_\oplus$ and $R_p = 4.3$ $R_\oplus$ from Deming et al. (2007). The internal structure in Figure 3 shows how three planets with very different internal compositions can have the same total mass and radius. We first explore a planet similar to our fiducial model: a 22.2 $M_\oplus$ solid planet with Earth-like iron/rock mass ratio (30/70), $T_{\rm eff} = 30$ K, and $T_{\rm eq} = 600$ K, in rough agreement with the orbital parameters (assuming an albedo of 0.1). By adding $\sim$3.2 $M_\oplus$ of hydrogen-helium to the 19.0 $M_\oplus$ solid planet, we are able to reproduce GJ 436b's radius. We note that the mass of gas is 15% of the solid mass, likely too much to have originated from outgassing, and so capture must be at least partially invoked to explain such a massive atmosphere. The second composition for GJ 436b we considered is for water worlds, one with a 50% water mantle (by mass) and 50% silicate core, and another with 90% water mantle and a 10% silicate core. These planets also need some H/He to match the known radius, 12% and 8% by mass, respectively. The third model approximates planets with atmospheres created from outgassing, considering an extreme scenario in which all of the available water has oxidized iron, leaving a 100% $(Mg,Fe)SiO_3$ solid planet core. To match the observed radius, a 22.2 $M_\oplus$ planet requires $\sim$2.2 $M_\oplus$ of H alone, a case that assumes no initial trapping and subsequent outgassing of He. The pure-hydrogen atmosphere is 10% of the mass of the solid planet. This is above the theoretical maximum of outgassing based on observed abundances of metallic iron in chondritic meteorites from our solar system (see L. Elkins-Tanton et al., in preparation). Although not an exhaustive study, the range of interior compositions illustrates the variety of possibilities, although all models require some H/He.

While our study is preliminary, we make the robust point that H-rich thick atmospheres will confuse the interpretation of planets based on a measured mass and radius. This point is independent of the uncertainties retained by our model including $T_{\rm eq}$, $T_{\rm eff}$, the mass fraction of H/He, and the mixing ratio of H and He. We find that the identification of water worlds based on the mass-radius relationship alone is impossible unless a significant gas layer can be ruled out by other means. Spectroscopy is the most likely means, and may become routine with transit transmission and emission spectroscopy, and eventually with spectroscopy by direct imaging.

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